

# Physics of Coulomb Corrections in Hanbury-Brown Twiss Interferometry in Ultrarelativistic Heavy Ion Collisions

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## Abstract

We discuss the elementary physics of the final state Coulomb interactions in Hanbury-Brown Twiss interferometry, showing – with explicit comparison to E877 data for  $\pi^+\pi^-$  and  $\pi^\pm p$  – that the Coulomb corrections in the pair correlation function can be well understood in terms of simple classical physics. We connect the classical picture with descriptions in terms of Coulomb wave functions, and investigate the influence of the “central” Coulomb potential on the pair correlation function.

## 1 Introduction

Hanbury-Brown Twiss interferometry of identical mesons has become an important probe of the evolving geometry of the collision volume in ultrarelativistic heavy-ion collisions [1-3]. The quantitative interpretation of the results depends critically on understanding the role of Coulomb interactions of the detected pairs of particles with each other, as well as the Coulomb interactions of the pair with the system of remaining particles. In this note we focus on the elementary physics of these processes, deferring detailed calculations to later publications.

The simplest form of Coulomb correction is inclusion of the Gamow factor – the square of the relative Coulomb wave function,  $\psi_C(0)$ , of the produced pair at zero separation – a

procedure which is followed in many analyses [4, 5]. The assumption is that the produced pair of identical particles is made in a relative Coulomb state (at zero separation), and the amplitude for doing so is thus modified from the bare amplitude by the factor  $\psi_C(0)$ ; non-relativistically

$$\psi_C(0) = \left( \frac{2\pi\eta}{e^{2\pi\eta} - 1} \right)^{1/2}, \quad (1)$$

where the dimensionless parameter  $\eta$  is given by

$$\eta = \frac{zz'e^2}{v_{rel}} = \frac{zz'\alpha}{v_{rel}/c}, \quad (2)$$

for a pair of particles of charges  $ze$  and  $z'e$  with relative momentum  $\vec{q} = (\vec{p} - \vec{p}')/2$  and relative velocity  $v_{rel} = q/m_{red}$ , with reduced mass  $m_{red} = m/2$  for two particles of mass  $m$ . [We consider particles with  $|z| = 1$  here.] The Coulomb-corrected rate of production is then inferred to be the measured rate divided by  $|\psi_C(0)|^2$ . For pairs of the same charge, the Gamow correction suppresses the probability of production at small  $q$ , by a factor tending at small  $q$  to  $2\pi\eta e^{-2\pi\eta}$ , while enhancing the probability of production for opposite sign pairs by a factor tending to  $2\pi|\eta|$  at small  $q$ .

Correcting for Coulomb effects by taking the relative Coulomb wave function at the origin is not physically correct in heavy ion collisions. As noted in Refs. [6, 7] (see also [8-11]), taking the finite size of the source size into account can produce significant effects. The standard Gamow correction assumes that the separation of the particles of the pair at creation is small compared with the (zero angular momentum) *classical turning point*,  $r_t$ , defined by  $q^2/2m_{red} = e^2/r_t$ . However, for pions,  $r_t \simeq (200 \text{ fm})/q^2$ , where  $q$  is measured in MeV/c; for  $q \sim 10 \text{ MeV/c}$ , a typical minimum value,  $r_t$  is only 2 fm, and smaller for larger  $q$ . Since  $r_t$  is much smaller than the characteristic heavy ion radius, most of the pairs of particles observed in a heavy ion collision are made at relative separations well outside their classical turning points.

We note that for typical  $q$ , the three length scales in the problem – the turning point, the particle wavelength, and the two-particle Bohr radius,  $a_0 = 1/m_{red}e^2$  ( $= 387 \text{ fm}$  for  $\pi\pi$  and  $222 \text{ fm}$  for  $\pi p$ ) – are cleanly separated:

$$r_t : 1/q : a_0 = 2 : a_0 q : (a_0 q)^2. \quad (3)$$

For  $\pi\pi$  (or  $\pi p$ ),  $a_0 q = 1/|\eta| = 1.96$  (or  $1.13$ )  $q/(\text{Mev/c}) \gg 1$ . The classical turning point is the relevant length scale here for Coulomb effects (not, as suggested in Refs. [2, 6], the two-particle Bohr radius).

Furthermore, in the presence of many produced particles, the relative Coulomb interaction of a pair is highly screened, which also decreases effects of Coulomb suppression. The

motion of the particles in the pair is strongly affected by their interactions with the plasma of other particles, and the mutual Coulomb interaction of the pair becomes dominant only when the pair has sufficiently separated from the other particles in the system that there is small probability of finding other particles between the particles in the pair. (See Ref. [12] for a recent investigation of screening effects in the context of a particular model for high particle multiplicities.)

The major effect of the Coulomb interaction between the particles of the pair, at distances large compared with  $r_t$ , is to accelerate them relative to each other. Particles of the same charge are accelerated to larger relative momenta, thus depressing the observed distribution at small  $q$ , while particles of opposite charge are reduced in relative momentum in the final state, which builds up the distribution at small  $q$ . Although these effects are qualitatively similar those produced by the Gamow correction, they are quantitatively rather different.

Our main focus in this paper is on a simple schematic model for effects of Coulomb interactions. As we shall see, correcting for the Coulomb final state interaction of non-identical pairs enables one to extract important information contained in the measured correlation function, about the spatial and temporal size of the emitting source. The same detailed information about the Coulomb final state interaction is also important to correct correlation functions of identical particles for Coulomb effects.

## 2 Toy model

We construct a greatly simplified model to take the screening and acceleration effects into account by neglecting the Coulomb interaction between the pair for separations less than an initial radius  $r_0$ , and for separations greater than  $r_0$  including only the relative Coulomb interaction. Since the relative motion is in the classical region, conservation of energy of the pair implies that the final observed relative momentum  $q$  is related to the initial momentum of the pair  $q_0$  at  $r_0$  by (see, e.g., [5, 13])

$$\frac{q^2}{2m_{red}} = \frac{q_0^2}{2m_{red}} \pm \frac{e^2}{r_0}, \quad (4)$$

where the upper sign is for particles of like charge, and the lower for particles of opposite charge. For example, for pions with  $r_0 = 10$  fm,  $q^2 = q_0^2 \pm 20(\text{MeV}/c)^2$ .

How does the acceleration affect the measured correlation function? In a heavy ion collision the distribution of singles is given in terms of particle creation and annihilation operators by

$$n(\vec{p}) = \langle a^\dagger(\vec{p}) a(\vec{p}) \rangle, \quad (5)$$

while the distribution of pairs of particles of momenta  $\vec{p}$  and  $\vec{p}'$  is given by

$$n_2(\vec{p}, \vec{p}') = \langle a^\dagger(\vec{p}) a^\dagger(\vec{p}') a(\vec{p}') a(\vec{p}) \rangle. \quad (6)$$

Interferometry experiments measure the pair correlation function

$$C(\vec{q}) = \frac{\{n_2(\vec{p}, \vec{p}')\}}{\{n(\vec{p})n(\vec{p}')\}}, \quad (7)$$

where the braces in the numerator denote an average over the total momentum  $\vec{P}$  for an ensemble of pairs from the same events at fixed relative momentum  $\vec{q} = (\vec{p} - \vec{p}')/2$ , and in the denominator they denote an average over particle pairs drawn from different events. Note that the 4-vector product  $q \cdot P$  vanishes identically, implying that the component of the relative momentum along the pair direction  $q_{\parallel}$  equals  $(E_1 - E_2)/2\beta$ , where  $\beta$  is the pair velocity and  $(E_1 - E_2)/2$  is the relative pair energy.

Since the Coulomb interaction conserves particles and the total momentum of the pair, the final distribution  $n_2(\vec{p}, \vec{p}')$  of pairs of relative momenta  $q$  is thus given in terms of the initial distribution of pairs,  $n_2^0(\vec{p}_0, \vec{p}_0')$ , by

$$n_2(\vec{p}, \vec{p}') d^3q = n_2^0(\vec{p}_0, \vec{p}_0') d^3q_0 \quad (8)$$

Equation (4), with changes in relative angles ignored, yields the familiar Jacobian (see, e.g., [5])  $d^3q_0/d^3q = q_0/q$ . We can assume to good accuracy that the Coulomb interactions between pairs of particles negligibly affect the singles distributions in the denominator of (7). Then

$$C(\vec{q}) = \frac{q_0}{q} C_0(\vec{q}_0) = \left(1 \mp \frac{2m_{red}e^2}{r_0q^2}\right)^{1/2} C_0(\vec{q}_0). \quad (9)$$

To illustrate this toy model, we compare in Fig. 1 the predictions of Eq. (9) with E877 data for the  $\pi^+\pi^-$ ,  $\pi^-p$ , and  $\pi^+p$  systems produced in Au+Au collisions at the AGS [14], assuming that the bare correlation function  $C_0$  equals unity. Shown in this figure as dotted lines are the results of the toy model for  $r_0 = 3$  fm (rightmost curve), 9 fm, and 15 fm (leftmost curve), along with standard Gamow correction (solid line). Except at very small relative momenta  $q \lesssim 10$  MeV/c, where effects due to the finite momentum resolution of the experiment become visible in the data, the model gives a good account of the data for  $r_0$  in the range of 9 - 15 fm. By contrast, the Gamow factor considerably overpredicts the data for all  $q$  values shown here. As we see, the raw correlation data for non-identical particles contains information about the mean separation of pairs when screening effects become negligible, summarized in the toy model by the (possibly  $q$  dependent) parameter

$r_0$ . Then we show the Coulomb correction factor deduced from Eq. (9) for  $\pi^+\pi^+$  in Fig. 2a and  $\pi^-\pi^-$  in Fig. 2b, for the same range of  $r_0$  (as in Fig. 1, the rightmost curve corresponds to  $r_0 = 3$  fm). Again we see that use of the Gamow factor implies a correction which differs significantly from that of the toy model.

With the initial radius  $r_0$  extracted from the unlike-sign data, one can then construct the Coulomb correction for like-sign particles<sup>1</sup>. Dividing the “raw” E877 data by the toy model correction factor, with  $r_0 = 15$  fm, we obtain the correlation function for like-sign pions (crosses) shown in Fig. 3a for  $\pi^+\pi^+$  and Fig. 3b for  $\pi^-\pi^-$ , which also show the correlation function (vertical bars) derived by making the standard Gamow correction. Using the Gamow factor instead of the proper Coulomb correction leads to a correlation function which is  $\sim 30\%$  wider, implying a correspondingly reduced radius parameter. Furthermore, the shape of the “Gamow-corrected” correlation function has considerable non-gaussian tails in the range  $30 < q < 80$  MeV/c. These tails do not exist in the raw correlation function and obscure the interpretation of the data.

We note also that the procedure described here for Coulomb corrections is not restricted to one-dimensional correlation functions. Since the Coulomb correction depends only on the magnitude of the relative momentum of the pair and not on its orientation, one should, for example, in an multi-dimensional analysis in terms of  $q_{out}$ ,  $q_{side}$ , and  $q_{long}$ , apply the correction for, say, each bin of  $q_{out}$ ,  $q_{side}$ ,  $q_{long}$  separately, i.e., before projection onto the particular variable of interest.

### 3 Connection with quantum-mechanical description

The physics of the toy model above is contained in the Coulomb wave function describing the propagation of the pair. To recall how this works we suppress the total momentum of the pair, and focus only on the relative momentum. In the absence of Coulomb interactions the number of pairs of relative momentum  $\vec{q}$  is given by

$$N_0(\vec{q}) = \int d^3r d^3r' e^{i\vec{q} \cdot (\vec{r} - \vec{r}')} \langle J^\dagger(\vec{r}) J(\vec{r}') \rangle, \quad (10)$$

where  $J(\vec{r})$  is the amplitude for creating a pair at separation  $\vec{r}$ , and the brackets denote an average over the event. In the presence of Coulomb interactions we have instead

$$N(\vec{q}) = \int d^3r d^3r' \psi_C(\vec{r}) \psi_C^*(\vec{r}') \langle J^\dagger(\vec{r}) J(\vec{r}') \rangle, \quad (11)$$

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<sup>1</sup>It is, in general, inadequate to correct the like-sign data with the inverse of the unlike-sign Coulomb correction. For  $\pi - \pi$  correlations this approximation is a rather good, except at very small  $q$ ; it fails, however, for correlations among heavier particles.

where  $\psi_C(\vec{r}) = \psi_C(0)_1F_1(-i\eta; 1; i(qr - \vec{q} \cdot \vec{r}))$  is the relative Coulomb wave function for a pair of relative momentum  $\vec{q}$  at infinity.

Pairs of low relative momentum have relatively low angular momentum, e.g., a pair produced at 10 fm separation with relative momentum 20 MeV/c can have at most one unit of relative angular momentum. Thus only the low partial wave components of the Coulomb wave function enter Eq. (11) with appreciable probability. We consider here just s-waves, for which we employ the WKB approximation to write the Coulomb wave function at radius  $r$  outside the classical turning point as<sup>2</sup>

$$\psi(r) \simeq \frac{1}{rp(r)^{1/2}q^{1/2}} \sin \phi(r), \quad (12)$$

where the local relative momentum, measuring the rate of change of phase,  $\phi$ , of the wave function, is given by

$$p(r) = \frac{d\phi}{dr} = \left( q^2 \mp \frac{2m_{red}e^2}{r} \right)^{1/2}. \quad (13)$$

(Equation (12), with  $\ell$ -dependent  $\phi(r)$  holds as well for higher partial waves,  $\ell > 0$ .) The normalization of (12) agrees with (1) as  $r \rightarrow 0$ , while as  $r \rightarrow \infty$ , the Coulomb wave function approaches

$$\psi(r) = \frac{1}{qr} \sin(qr - \eta \ln 2qr + \delta_0). \quad (14)$$

The results of the toy model follow if we assume that the source correlation function  $\langle J^\dagger(\vec{r})J(\vec{r}') \rangle$  is localized in both  $r$  and  $r'$  around  $r_0$ . The correlation function for s-waves (denoted by superscript  $s$ ), the absence of Coulomb, is

$$N_0^s(p) = \int d^3r d^3r' \frac{\sin pr}{pr} \frac{\sin pr'}{pr'} \langle J^\dagger(\vec{r})J(\vec{r}') \rangle; \quad (15)$$

then since in the region of any radius  $r$  outside the turning point the Coulomb wave function behaves locally as a free particle s-wave of momentum  $p(r)$ , the correlation function is given by

$$N^s(q) \approx \frac{p(r_0)}{q} N_0^s(p(r_0)) \quad (16)$$

where the factor  $p(r_0)/q$  arises from the denominators in Eq. (12) and (15). Consequently,  $C(q) \simeq C_0(p(r_0))p(r_0)/q$ , the result in Eq. (9) with  $q_0 = p(r_0)$ .

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<sup>2</sup>The WKB approximation for the s-wave is quite good outside the collision volume for the parameters encountered in HBT interferometry in ultrarelativistic collisions. The condition for validity of the approximation is  $|\partial p(r)/\partial r| \ll p(r)^2$ , which for  $r \ll a_0$ , the region of interest, becomes the restriction,  $r \gtrsim 3/q^{3/2}a_0^{1/2}$ . For  $\pi\pi$  (or  $\pi p$ ) pairs with  $q > 20$  MeV/c, WKB is reasonable for  $r$  down to  $\sim 5$  fm (or  $\sim 6$  fm).

With the connection between the toy model and the Coulomb wave function established we can now generalize the picture to extend to smaller values of the source radius  $r_0$ . In general, the effect of the Coulomb interactions depends on the detailed structure of the source correlation function  $\langle J^\dagger(\vec{r})J(\vec{r}') \rangle$ ; we expect this correlation function to be approximately of the form

$$\langle J^\dagger(\vec{r})J(\vec{r}') \rangle \approx S(\vec{r}, \vec{r}')f(\vec{r}' - \vec{r}); \quad (17)$$

$S$ , which defines the spatial region of the source, varies as a function of its arguments on a scale of the size of the emitting region, and the Fourier transform of  $f$  defines the bare distribution of pairs, viz.,

$$f(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} f(\vec{q}), \quad (18)$$

where, but for effects due to the finite size of the emitting region,  $N_0(\vec{q}) \sim f(\vec{q})$ .

For a first orientation we can identify the bare correlation of Eq. (18) with that experimentally observed for two-particle correlations in  $e^+e^-$  annihilation [15-18], which indicate an HBT “radius” consistently below 1 fm. Thus we expect  $f(\vec{r} - \vec{r}')$  to extend only over distances  $\lesssim 1$  fm, or equivalently the bare correlation to vary on momentum scales of several hundred MeV/c. Since typical source sizes from an analysis of pion correlations following ultra-relativistic nucleus-nucleus collisions are of the order of 5 - 10 fm [14, 3], it is a reasonable approximation to neglect the size of the bare correlation and replace  $f(\vec{r} - \vec{r}')$  in Eq. (17) by a delta function. We write

$$\langle J^\dagger(\vec{r})J(\vec{r}') \rangle \approx \delta(\vec{r} - \vec{r}')N_0S(r), \quad (19)$$

where  $S(r)$ , of unit strength, describes the distribution of initial pair radii. With (19) we find

$$C(q)/C_0(q) = N(q)/N_0 = \int d^3r |\psi_C(\vec{r})|^2 S(r) \quad (20)$$

(an equation tracing back to Ref. [6]).

To illustrate the transition from the Gamow correction to the toy model we take  $S(r)$  to have a simple normalized gaussian form of range  $r_0$ :  $S(r) = (2\pi)^{-3/2}r_0^{-3} \exp(-r^2/2r_0^2)$ . We show, in Fig. 4, for the  $\pi^+\pi^-$  system, the results of calculations of  $C(q)/C_0(q)$  using (20) for  $r_0 = 1, 5, 9$ , and 18 fm (dash-dot curves, the highest for  $r_0 = 1$  fm, and falling with increasing  $r_0$ ). As  $r_0 \rightarrow 0$ , the projection of the square of the Coulomb wave function onto the source  $S(r)$  converges to the standard Gamow correction (solid line).<sup>3</sup> For larger  $r_0$

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<sup>3</sup>For  $r_0 < 0.1$  fm (not shown in Fig. 4) the difference between the Gamow correction and a calculation with (20) is less than 0.5%.

values it rather quickly approaches the prediction of the toy model (shown here for an initial radius of 9 fm as a dotted curve), implying that, for pairs originating outside their classical turning point, the toy model provides an adequate and reasonably accurate description of the Coulomb effects.

## 4 The “central” Coulomb potential

We next turn to the question of the effects of the Coulomb interactions of the pair with the remaining particles. This is a difficult many-body problem, which we greatly simplify at this stage by assuming that the remaining particles can be described by a central Coulomb potential,  $Z_{eff}e^2/r$ , where in a central collision of nucleus A with nucleus B the effective charge  $Z_{eff}$  is of order of the total initial nuclear charge ( $Z_A + Z_B$ ). This central potential accelerates positive mesons away and slows down the negatives, effects described by the Coulomb wave functions for the potential. The final momentum of any particle is related to the initial momentum  $p_a$  at production point  $r_a$  by

$$\epsilon(p) = \epsilon(p_a) \pm \frac{Z_{eff}e^2}{r_a}. \quad (21)$$

where  $\epsilon(p) = (p^2 + m^2)^{1/2}$ . (While Coulomb effects for the relative momentum can be treated non-relativistically as in Eq. (4), the individual momenta are generally relativistic.) We ignore in this brief discussion quantum mechanical suppressions or enhancements of the amplitude for particle emission, as well as possible effects of angular changes in the individual particle orbits on the particle distributions. Then the single particle distribution is modified by the central potential, analogously to Eq. (8), by

$$n(\vec{p}) = n_0(\vec{p}_a) \frac{d^3 p_a}{d^3 p} = \frac{p_a \epsilon(p_a)}{p \epsilon(p)} n_0(\vec{p}_a). \quad (22)$$

Both the magnitude of the distribution as well as its argument are shifted.

Although the central potential shifts the singles distribution, it cannot introduce any correlations among emitted particles that have no initial correlation in the absence of the central potential, e.g., for different species or opposite charged pions, as one usually assumes. If in the absence of the central potential uncorrelated particles [ $C(q) = 1$ ] are emitted in independent free particle states, then in the presence of the potential they are emitted in Coulomb states for the central potential, but still  $n_2(\vec{p}, \vec{p}') = n(\vec{p})n(\vec{p}')$  and  $C(q)$  remains unity.

For particles that are initially correlated as a consequence of Bose-Einstein statistics,  $n_2(\vec{p}, \vec{p}')$  and  $n(\vec{p})n(\vec{p}')$  will be modified both by the Jacobians of the transformations from



initial to final momenta, and shifts of argument. However, in forming  $C(q)$ , the effects of the Jacobians in the numerator and denominator essentially cancel, and the primary effect is the shift in the arguments:

$$C(\vec{q}) = \frac{\{n_2(\vec{p}_a, \vec{p}_a')\}}{\{n(\vec{p}_a)n(\vec{p}_a')\}}. \quad (23)$$

Since positive particles are accelerated, the final momentum difference,  $\vec{q} = \vec{p} - \vec{p}'$ , of a positive pair will generally be larger in magnitude than it is initially, while for negative pairs the final momentum difference will generally be smaller. Thus we expect the central Coulomb potential to cause the size of collision volume extracted from positive pairs to be smaller than the actual size, and that from negative pairs larger than the actual size. As an illustration consider a pair of relativistic particles whose initial momenta  $\vec{p}_a$  and  $\vec{p}_a'$  are equal in magnitude to  $p_a$ , and final momenta  $\vec{p}$  and  $\vec{p}'$  equal in magnitude to  $p$ ; then

$$q = (p/p_a)q_a \simeq q_a \left( 1 \pm \frac{Z_{eff}e^2/r_a}{p_a} \right). \quad (24)$$

where the upper sign refers to both particles positively charged and the lower to both negatively charged. For  $Z \sim 150$ ,  $r_a \sim 7$  fm and  $p_a \sim 300$  MeV/c, the effect is an increase for positives (and a decrease for negatives) in the observed scale of  $C(q)$  and decrease (or increase) in the extracted radius of ten percent.

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## References

- [1] D. Boal, C.K. Gelbke, B. Jennings, Rev. Mod. Phys. **62** 553 (1990).
- [2] W. Bauer, C. Gelbke and S. Pratt, Ann. Rev. Nucl. Part. Sci. **42**, 77 (1992), and references therein.
- [3] B. V. Jacak, Nucl. Phys. **A590** 215c (1995).
- [4] M. Gyulassy, S. K. Kauffmann, and L. W. Wilson, Phys. Rev. **C20** 2267 (1979).
- [5] M. Gyulassy and S. K. Kauffmann, Nucl. Phys. **A362**, 503 (1981).
- [6] S. Pratt, Phys. Rev. **D33**, 72 (1986).
- [7] M. G. Bowler, Phys. Letters **B270**, 69 (1991).

- [8] S. Pratt, T. Csörgő, and J. Zimányi, Phys. Rev. **C42**, 2646 (1990).
- [9] T. Csörgő, J. Zimányi, J. Bondorf, H. Heiselberg, and S. Pratt, Phys. Letters **B241**, 301 (1990); T. Csörgő, J. Zimányi, J. Bondorf, and H. Heiselberg, Phys. Letters **B222**, 115 (1989);
- [10] D. Anchishkin and G. Zinovjev, Phys. Rev. **C51**, R2306 (1995).
- [11] M. Biyajima, T. Mizoguchi, T. Osada, and G. Wilk, preprint SULDP-1995-3, TU477, SINS-1995-1 (1995).
- [12] D.V. Anchishkin, W.A. Zajc, G.M. Zinovjev, preprint, Dec. 1995.
- [13] K. G. Libbrecht and S. E. Koonin, Phys. Rev. Letters. **43** 1581 (1979).
- [14] D. Miskowiec, E877 collaboration, Nucl. Phys. **A590** 473c (1995), and E877 collaboration preprint, June 1996.
- [15] P. D. Acton et al., OPAL collaboration, Phys. Lett. **B267** 143 (1991), Z. Phys. **C58** 207 (1993).
- [16] P. Abreu et al., DELPHI collaboration, Phys. Lett. **B286** 201 (1992).
- [17] D. Decamp et al., ALEPH collaboration, Z. Phys. **C54** 75 (1992).
- [18] G. Goldhaber, in *Hadronic Matter in Collision*, Proc. 2<sup>nd</sup> Int. Workshop on Local Equilibrium in Strong Interaction Physics, Santa Fe, 1986, P. Carruthers and D. Strottman, eds. (World Scientific, Singapore, 1986). p. 3.

## 5 Figure captions

FIG. 1. Comparison of the toy model, Eq. (9) for  $r_0 = 3$  fm (rightmost curve), 9 fm, and 15 fm (leftmost curve), with E877 data [14] for the systems  $\pi^+\pi^-$ ,  $\pi^-p$ , and  $\pi^+p$ , assuming a bare correlation function  $C_0 = 1$ . Solid line: Gamow correction.

FIG. 2. Coulomb correction, Eq. (9) for the systems  $\pi^+\pi^+$  and  $\pi^-\pi^-$  for the same range of  $r_0$  as in Fig. 1 (the rightmost curve corresponds to  $r_0 = 3$  fm).

FIG. 3. Toy model calculation of  $C(q)$  for like-sign pions (crosses), compared with the correlation function derived by making the standard Gamow correction (vertical bars).

FIG. 4. Transition from the toy model (dotted line, with  $r_0 = 9$  fm) to the Gamow correction (solid line) with decreasing source size, calculated from Eq. (20) (dash-dot curves). From highest to lowest dash-dot curves the source range  $r_0$  is 1, 5, 9, 18 fm. The data shown are for  $\pi^+\pi^-$ .

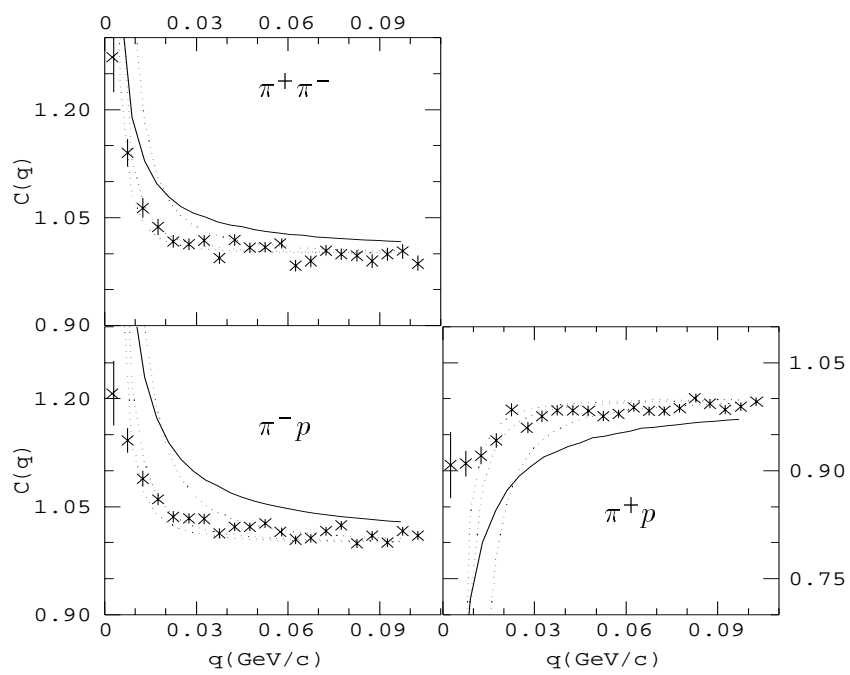


Fig. 1

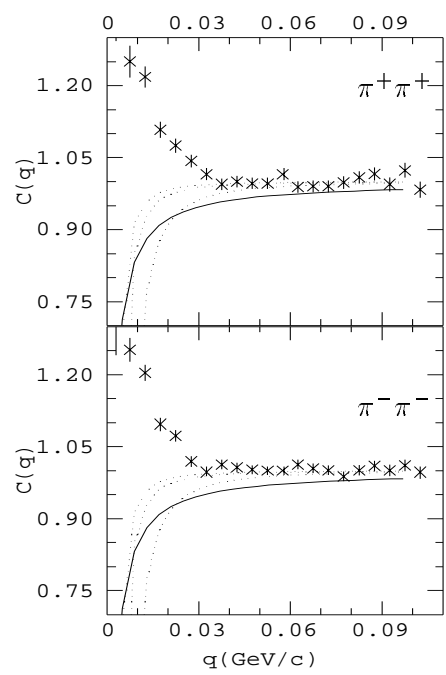


Fig. 2

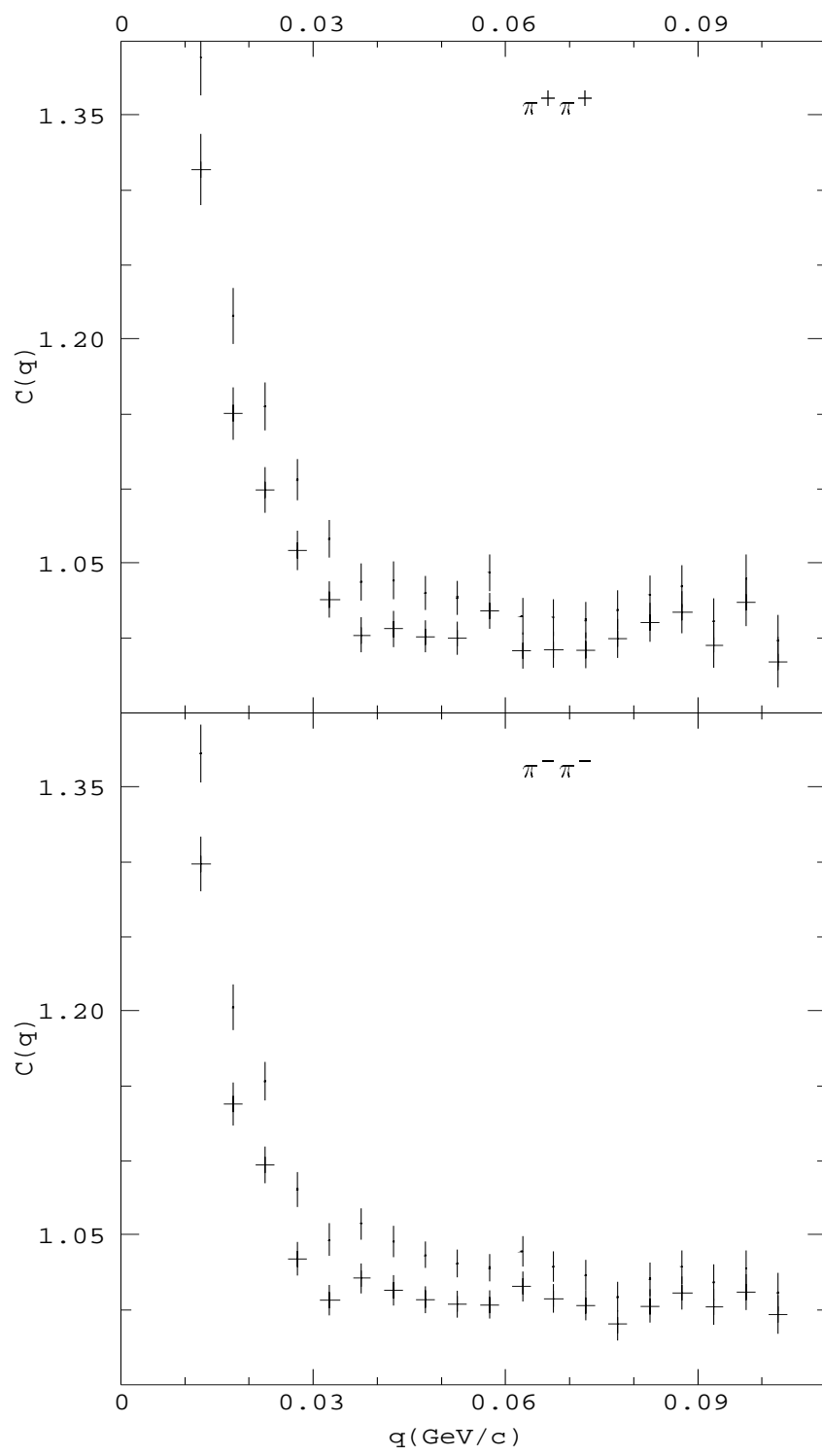


Fig. 3

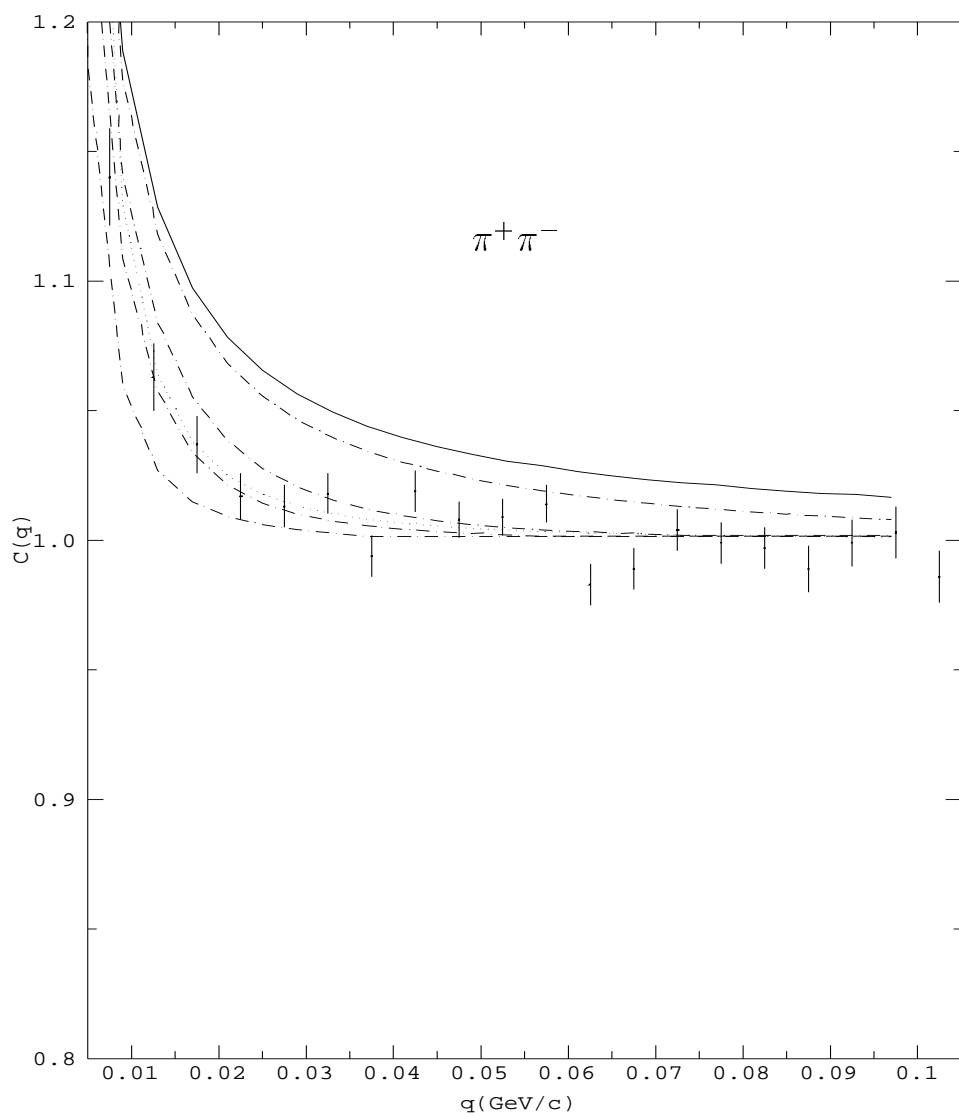


Fig. 4